EXAMPLE 1

A poll was taken of 2,020 workers in a city. The workers sampled were asked whether they take a 'duvet day', that is, call in sick, at least once a year when they simply need time to relax. 404 workers responded yes. Use these data to obtain a 95% confidence interval for the proportion, p, of all employees in the city who take a 'duvet day'.

Solution

Step 1: Calculate the standard error.

$$\text{SE} = \sigma_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \quad \text{where } n = 2{,}020$$

$$\text{and } \hat{p} = \frac{404}{2{,}020} = 0{\cdot}2.$$

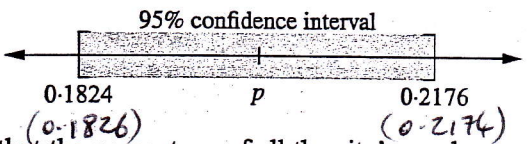
$$\text{Then } \sigma_{\hat{p}} = \sqrt{\frac{(0{\cdot}2)(1-0{\cdot}2)}{2{,}020}} = 0{\cdot}008899 = 0{\cdot}009.$$

Step 2: The margin of error, E, at the 95% confidence interval is given by

$$E = 1{\cdot}96\sigma_{\hat{p}} = 1{\cdot}96(0{\cdot}009) = 0{\cdot}0176.$$

Step 3: The 95% confidence interval for the true proportion, p, is then:

$$\hat{p} - E \leq p \leq \hat{p} + E$$
$$0{\cdot}2 - 0{\cdot}0176 \leq p \leq 0{\cdot}2 + 0{\cdot}0176$$
$$0{\cdot}1824 \leq p \leq 0{\cdot}2176$$
$$18{\cdot}24\% \leq p \leq 21{\cdot}76\%$$



We are 95% confident that the percentage of all the city's employees who take a duvet day is somewhere between 18·24% and 21·76%.

It is worth noting that the margin of error, E, is half the length of the confidence interval.