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Professional Development Service for Teachers

## Applied Maths Induction Workshop 1 - Accelerated Linear Motion - Solutions

## 2010 - Ordinary Level - Question 1

A car travels along a straight level road.
It passes a point $P$ at a speed of $12 \mathrm{~m} / \mathrm{s}$ and accelerates uniformly for 6 seconds to a speed of $30 \mathrm{~m} / \mathrm{s}$.

It then travels at a constant speed of $30 \mathrm{~m} / \mathrm{s}$ for 15 seconds.
Finally the car decelerates uniformly from $30 \mathrm{~m} / \mathrm{s}$ to rest at a point $Q$.
The car travels 45 metres while decelerating.

Find (i) the acceleration
(ii) the deceleration
(iii) $|P Q|$, the distance from $P$ to $Q$
(iv) the average speed of the car as it travels from $P$ to $Q$.

## Solution



Acceleration
$\left.\begin{array}{l}u=12 \\ v=30 \\ t=6\end{array}\right\} v=u+a t \Rightarrow a=\frac{v-u}{t} \Rightarrow a=\frac{30-12}{6} \Rightarrow a=3 \mathrm{~m} / \mathrm{s}^{2}$
Deceleration
$\left.\begin{array}{l}\left.\begin{array}{l}u=30 \\ v \\ =0 \\ s\end{array}\right\} 45\end{array}\right\} v^{2}=u^{2}+2 a s \Rightarrow a=\frac{v^{2}-u^{2}}{2 s} \Rightarrow a=\frac{0-900}{90} \Rightarrow a=-10 \mathrm{~m} / \mathrm{s}^{2}$
(iii) Distance from $P$ to $Q$ equals area under graph

$$
\begin{aligned}
& =(6 \times 12)+\left(\frac{1}{2} \times 6 \times 18\right)+(15 \times 30)+45 \\
& =621 \text { metres }
\end{aligned}
$$

(iv) Average Speed $=\frac{\text { Total Distance }}{\text { Total Time }}$

Firstly, must find time elapsed during deceleration phase:

$$
\begin{aligned}
& \left.\begin{array}{l}
u=30 \\
v=0 \\
a=-10
\end{array}\right\} v=u+a t \Rightarrow t=\frac{v-u}{a} \Rightarrow t=\frac{0-30}{-10} \Rightarrow t=3 \\
& \Rightarrow \quad \text { Total Time }=6+15+3=24 \text { seconds } \\
& \Rightarrow \quad \text { Average Speed }=\frac{621}{24} \simeq 26 \text { seconds }
\end{aligned}
$$

## 2007 - Ordinary Level - Question 1

A car travels from $p$ to $q$ along a straight level road.
It starts from rest at $p$ and accelerates uniformly for 5 seconds to a speed of $15 \mathrm{~m} / \mathrm{s}$.
It then moves at a constant speed of $15 \mathrm{~m} / \mathrm{s}$ for 20 seconds.
Finally the car decelerates uniformly from $15 \mathrm{~m} / \mathrm{s}$ to rest at $q$ in 3 seconds.
(i) Draw a speed-time graph of the motion of the car from $p$ to $q$.
(ii) Find the uniform acceleration of the car.
(iii) Find the uniform deceleration of the car.
(iv) Find $|p q|$, the distance from $p$ to $q$.
(v) Find the speed of the car when it is 13.5 metres from $p$.

## Solution

(i)

(ii) Acceleration

(iii) Deceleration

$$
\left.\begin{array}{l}
u=15 \\
v=0 \\
t=3
\end{array}\right\} v=u+a t \quad \Rightarrow \quad a=\frac{v-u}{t} \quad \Rightarrow \quad a=\frac{0-15}{3} \quad \Rightarrow a=-5 \mathrm{~m} / \mathrm{s}^{2}
$$

(iv) $|p q|=$ Area under graph $=\left[\frac{1}{2} \times 5 \times 15\right]+[20 \times 15]+\left[\frac{1}{2} \times 3 \times 15\right]$

$$
=37 \cdot 5+300+22 \cdot 5
$$

$$
=360 \mathrm{~m}
$$

(v) Reaches 13.5 m from $p$ during acceleration phase.

$$
\begin{aligned}
& u=0 \\
& \left.\begin{array}{l}
a=3 \\
s=13 \cdot 5
\end{array}\right\} v^{2}=u^{2}+2 a s \Rightarrow v=\sqrt{u^{2}+2 a s} \Rightarrow v=\sqrt{0+(2)(3)(13 \cdot 5)} \Rightarrow v=9 \mathrm{~m} / \mathrm{s} \\
& \hline
\end{aligned}
$$

## 2007 - Higher Level - Question 1(b)

A train accelerates uniformly from rest to a speed $v \mathrm{~m} / \mathrm{s}$.

It then continues at this speed for a period of time and then decelerates uniformly to rest.

In travelling a total distance $d$ metres the train accelerates through a distance $p d$ metres and decelerates through a distance $q d$ metres, where $p<1$ and $q<1$.
(i) Draw a speed-time graph for the motion of the train.
(ii) If the average speed of the train for the whole journey is $\frac{v}{p+q+b}$, find the value of $b$.

## Solution



Average Speed $=\frac{\text { Total Distance }}{\text { Total Time }}$
We know that the total distance is $d$. We therefore need to find the time for each section of the journey.

## Acceleration

Area under graph $=p d \quad \Rightarrow \quad \frac{1}{2}\left(t_{1}\right)(v)=p d \quad \Rightarrow \quad t_{1}=\frac{2 p d}{v}$

## Constant Speed

Area under graph $=d-p d-q d \quad \Rightarrow \quad t_{2} v=d-p d-q d \quad \Rightarrow \quad t_{2}=\frac{d-p d-q d}{v}$

## Deceleration

Area under graph $=q d \quad \Rightarrow \quad \frac{1}{2}\left(t_{3}\right)(v)=q d \quad \Rightarrow \quad t_{3}=\frac{2 q d}{v}$
$\Rightarrow \quad$ Total Time $=\frac{2 p d}{v}+\frac{d-p d-q d}{v}+\frac{2 q d}{v}=\frac{2 p d+d-p d-q d+2 q d}{v}=\frac{d+p d+q d}{v}$
$\Rightarrow \quad$ Average Speed $=\frac{d}{\underline{d+p d+q d}}=\frac{\not \Delta v}{\not \lambda(1+p+q)}=\frac{v}{p+q+1}$

But Average Speed $=\frac{v}{p+q+b} \quad \Rightarrow \quad b=1$

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1999 - Higher Level - Question 1(b)
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A particle travels in a straight line with constant acceleration $f$ for $2 t$ seconds and covers 15 metres. The particle then travels a further 55 metres at constant speed in $5 t$ seconds. Finally the particle is brought to rest by a constant retardation $3 f$.
(i) Draw a speed-time graph for the motion of the particle.
(ii) Find the initial velocity of the particle in terms of $t$.
(iii) Find the total distance travelled in metres, correct to two decimal places.

## Solution



Constant Speed

$$
5 t v=55 \quad \Rightarrow \quad v=\frac{11}{t}
$$

## Acceleration

$$
\begin{align*}
& \left.\begin{array}{l}
u=u \\
\begin{array}{l}
a=f \\
t
\end{array}=2 t \\
s=15 \\
v=\frac{11}{t}
\end{array}\right\} \text { There is enough information here for two equations in } u \text { and } f \\
& \begin{aligned}
s & =u t+\frac{1}{2} a t^{2}
\end{aligned} \\
& \begin{aligned}
& v=u+a t \quad 15=2 u t+2 f t^{2} \quad \Rightarrow \quad \frac{11}{t}=u+2 f t \\
& \Rightarrow \quad 11=u t+2 f t^{2} \\
& \Rightarrow \quad 11-u t=15-2 u t
\end{aligned} \\
& \\
& \\
& \Rightarrow \quad u t=4 \Rightarrow \quad f=\frac{11-u t}{2 t^{2}}
\end{align*}
$$

(iii) Need to find distance travelled during deceleration:

$$
\left.\begin{array}{rl}
\begin{array}{l}
u=\frac{11}{t} \\
v=0 \\
a=-3 f
\end{array}
\end{array}\right\} v^{2}=u^{2}+2 a s \quad \Rightarrow \quad s=\frac{v^{2}-u^{2}}{2 a} \quad \Rightarrow \quad s=\frac{0-\frac{121}{t^{2}}}{-6 f}, ~ \begin{array}{ll} 
& \Rightarrow \quad s=\frac{121}{6 f t^{2}}
\end{array}
$$

But $f=\frac{11-u t}{2 t^{2}} \Rightarrow f=\frac{11-\left(\frac{4}{t}\right) t}{2 t^{2}} \Rightarrow f=\frac{7}{2 t^{2}}$

$$
\begin{aligned}
& \Rightarrow \quad s=\frac{121}{6\left(\frac{7}{2 \vdash^{\not ㇒}}\right) \vdash^{\not ㇒}} \\
& \Rightarrow \quad s=\frac{121}{21} \simeq 5.76 \mathrm{~m}
\end{aligned}
$$

$\Rightarrow \quad$ Total distance travelled $\simeq 15+55+5 \cdot 76=75 \cdot 76 \mathrm{~m}$

## 2009 - Higher Level - Question 1(b)

A train accelerates uniformly from rest to a speed $v \mathrm{~m} / \mathrm{s}$ with uniform acceleration $f \mathrm{~m} / \mathrm{s}^{2}$.

It then decelerates uniformly to rest with uniform retardation $2 f \mathrm{~m} / \mathrm{s}^{2}$.

The total distance travelled is $d$ metres.
(i) Draw a speed-time graph for the motion of the train.
(ii) If the average speed of the train for the whole journey is $\sqrt{\frac{d}{3}}$, find the value of $f$.

## Solution


$t_{1}: t_{2}=2 f: f=2: 1 \quad \Rightarrow \quad t_{1}=\frac{2}{3} T \quad$ and $\quad t_{2}=\frac{1}{3} T$

## Acceleration

$\left.\begin{array}{l}u=0 \\ a=f \\ t=\frac{2}{3} T\end{array}\right\} v=u+a t \quad \Rightarrow \quad v=\frac{2}{3} f T$
$\begin{aligned} & d=\text { Area under graph } \Rightarrow \quad d=\frac{1}{2} T\left(\frac{2}{3} f T\right) \quad \Rightarrow \quad d=\frac{f T^{2}}{3} \\ & \text { Average Speed }=\frac{\text { Total Distance }}{\text { Total Time }}=\frac{d}{T}=\frac{f T}{3}\end{aligned}$
But, Average Speed $=\sqrt{\frac{d}{3}} \Rightarrow \quad \frac{f T}{3}=\sqrt{\frac{d}{3}} \quad \Rightarrow \quad \frac{f^{2} T^{2}}{9}=\frac{d}{3} \quad \Rightarrow \quad d=\frac{f^{2} T^{2}}{3}$
$\Rightarrow \quad \frac{f T^{2}}{3}=\frac{f^{2} T^{2}}{3} \Rightarrow f=f^{2} \quad \Rightarrow \quad f(f-1)=0 \Rightarrow f=0 \quad f=1$

## 2006 - Higher Level - Question 1(a)

A lift starts from rest. For the first part of its descent it travels with uniform acceleration $f$. It then travels with uniform retardation $3 f$ and comes to rest. The total distance travelled is $d$ and the total time taken is $t$.
(i) Draw a speed-time graph for the motion.
(ii) Find $d$ in terms of $f$ and $t$.

## Solution


$t_{1}: t_{2}=3: 1 \quad \Rightarrow \quad t_{1}=\frac{3}{4} t \quad$ and $\quad t_{2}=\frac{1}{4} t$

## Acceleration

$$
\begin{aligned}
& \left.\begin{array}{l}
u=0 \\
\begin{array}{l}
a=f \\
t=\frac{3}{4} t
\end{array}
\end{array}\right\} v=u+a t \quad \Rightarrow \quad v=\frac{3}{4} f t \\
& d=\text { Area under graph } \Rightarrow \quad d=\frac{1}{2} t\left(\frac{3}{4} f t\right) \quad \Rightarrow \quad d=\frac{3 f t^{2}}{8}
\end{aligned}
$$

## 2008 - Higher Level - Question 1(a)

A ball is thrown vertically upwards with an initial velocity of $39 \cdot 2 \mathrm{~m} / \mathrm{s}$.

Find (i) the time taken to reach the maximum height
(ii) the distance travelled in 5 seconds.

## Solution

$\left.\begin{array}{l}u=39 \cdot 2 \\ \left.\begin{array}{l}a \\ = \\ v\end{array}\right\}-9 \cdot 8\end{array}\right\} v=u+a t \Rightarrow t=\frac{v-u}{a} \Rightarrow t=\frac{0-39 \cdot 2}{-9 \cdot 8} \Rightarrow t=4 \mathrm{~s}$
(ii) First five seconds: 4 seconds upwards, 1 second downwards.

Upwards

Downwards
$\left.\begin{array}{l}u=0 \\ a=9 \cdot 8 \\ t=1\end{array}\right\} s=u t+\frac{1}{2} a t^{2} \quad \Rightarrow \quad s=(0)(1)+\frac{1}{2}(9 \cdot 8)(1) \quad \Rightarrow \quad s=4 \cdot 9, ~$
$\Rightarrow \quad$ Distance travelled in 5 seconds $=78 \cdot 4+4 \cdot 9=83 \cdot 3 \mathrm{~m}$

## 2002 - Higher Level - Question 1(a)

A stone is thrown vertically upwards under gravity with a speed of $u \mathrm{~m} / \mathrm{s}$ from a point 30 metres above the horizontal ground.
The stone hits the ground 5 seconds later.
(i) Find the value of $u$.
(ii) Find the speed with which the stone hits the ground.

## Solution


(i)

$$
\begin{aligned}
& \Rightarrow u=\frac{-60+245}{10} \Rightarrow u=18 \cdot 5 \mathrm{~m} / \mathrm{s}
\end{aligned}
$$

(ii)


## 2008 - Higher Level - Question 1(b)

Two particles $P$ and $Q$, each having constant acceleration, are moving in the same direction along parallel lines. When $P$ passes $Q$ the speeds are $23 \mathrm{~m} / \mathrm{s}$ and $5.5 \mathrm{~m} / \mathrm{s}$, respectively. Two minutes later $Q$ passes $P$, and $Q$ is then moving at $65.5 \mathrm{~m} / \mathrm{s}$.

Find (i) the acceleration of $P$ and the acceleration of $Q$
(ii) the speed of $P$ when $Q$ overtakes it
(iii) the distance $P$ is ahead of $Q$ when they are moving with equal speeds.

## Solution

(i) Motion of $P$ :

$$
\left.\begin{array}{l}
u=23 \\
a=a_{1} \\
t=120
\end{array}\right\}
$$

Motion of $Q$ :

$$
\begin{aligned}
& \left.\begin{array}{l}
u=5 \cdot 5 \\
v=65 \cdot 5 \\
a=a_{2} \\
t=120
\end{array}\right\} v=u+a t \Rightarrow \quad a=\frac{v-u}{t} \\
& \Rightarrow \quad a_{2}=\frac{65 \cdot 5-5 \cdot 5}{120} \Rightarrow a_{2}=\frac{1}{2} \mathrm{~m} / \mathrm{s}^{2}
\end{aligned}
$$

Overtaking occurs when $s_{1}=s_{2}$

$$
\begin{aligned}
& s=u t+\frac{1}{2} a t^{2} \\
\Rightarrow \quad s & =(23)(120)+\frac{1}{2} a_{1}\left(120^{2}\right) \\
\Rightarrow \quad s & =2760+7200 a_{1}
\end{aligned}
$$

$$
\Rightarrow \quad 2760+7200 a_{1}=4260
$$

$$
\Rightarrow \quad a_{1}=\frac{5}{24} \mathrm{~m} / \mathrm{s}^{2}
$$

(ii) Motion of $P$ :

$$
\begin{aligned}
& u=23 \\
& \left.\begin{array}{l}
a=\frac{5}{24} \\
t=120
\end{array}\right\} v=u+a t \quad \Rightarrow \quad v=23+\frac{5}{24}(120) \quad \Rightarrow \quad v=48 \mathrm{~m} / \mathrm{s} \\
& \hline
\end{aligned}
$$

(iii) Need to find the time at which $v_{1}=v_{2}$ and then find the difference between $s_{1}$ and $s_{2}$ at this time.
Motion of $P$
Motion of $Q$
$\left.\begin{array}{l}u=23 \\ a=\frac{5}{24} \\ t=t\end{array}\right\} v=u+a t$

$$
v_{1}=23+\frac{5}{24} t
$$

$$
\begin{aligned}
& \left.\begin{array}{l}
u=5 \cdot 5 \\
a=\frac{1}{2} \\
t=t
\end{array}\right\} v=u+a t \\
& v_{2}=5 \cdot 5+\frac{1}{2} t
\end{aligned}
$$

$$
\begin{aligned}
& v_{1}=v_{2} \\
& \Rightarrow \quad 23+\frac{5}{24} t=5 \cdot 5+\frac{1}{2} t \quad \text {...multiply by } 24 \\
& \Rightarrow \quad 552+5 t=132+12 t \\
& \Rightarrow \quad 7 t=420 \\
& \Rightarrow \quad t=60 \\
& s_{1}=(23)(60)+\frac{1}{2}\left(\frac{5}{24}\right)\left(60^{2}\right) \quad s_{2}=(5 \cdot 5)(60)+\frac{1}{2}\left(\frac{1}{2}\right)\left(60^{2}\right) \\
& s_{1}=1755 \\
& s_{2}=1230 \\
& 1755-1230=525 \\
& \Rightarrow \quad P \text { is } 525 \text { metres ahead of } Q \text { when they are travelling with equal speeds. }
\end{aligned}
$$

## 2005 - Higher Level - Question 1(a)

Car $A$ and car $B$ travel in the same direction along a horizontal straight road.
Each car is travelling at a uniform speed of $20 \mathrm{~m} / \mathrm{s}$.
Car $A$ is at a distance of $d$ metres in front of car $B$.
At a certain instant car $A$ starts to brake with a constant retardation of $6 \mathrm{~m} / \mathrm{s}^{2}$.
0.5 s later car $B$ starts to brake with a constant retardation of $3 \mathrm{~m} / \mathrm{s}^{2}$.

Find (i) the distance travelled by car $A$ before it comes to rest.
(ii) the minimum value of $d$ for car $B$ not to collide with car $A$.

## Solution

(i) $\operatorname{Car} A$

| $\left.\begin{array}{l}u=20 \\ a \\ =-6 \\ v\end{array}\right\} v^{2}=u^{2}+2 a s \Rightarrow s=\frac{v^{2}-u^{2}}{2 a} \Rightarrow s=\frac{0-400}{-12} \Rightarrow s=\frac{100}{3} \mathrm{~m}$ |
| :--- |

(i) $\operatorname{Car} B$

First 0.5 seconds
Afterwards

$$
\begin{aligned}
& \left.\begin{array}{l}
u=20 \\
t=0 \cdot 5 \\
a=0
\end{array}\right\} s=u t+\frac{1}{2} a t^{2} \\
& \Rightarrow \quad s=(20)(0 \cdot 5) \\
& \Rightarrow \quad s=10
\end{aligned}
$$

$$
\left.\begin{array}{l}
u=20 \\
a=-3 \\
v=0
\end{array}\right\} s=\frac{v^{2}-u^{2}}{2 a}
$$

$$
\Rightarrow \quad s=\frac{0-400}{-6}
$$

$$
\Rightarrow \quad s=\frac{200}{3}
$$

$\Rightarrow \quad$ Overall, it takes car $B\left(10+\frac{200}{3}\right)=\frac{230}{3}$ metres to stop.
$\Rightarrow \quad d$ must be at least $\frac{230}{3}-\frac{100}{2}=\frac{130}{3}$ metres

## 2008 - Ordinary Level - Question 1

Four points $a, b, c$ and $d$ lie on a straight level road.
A car, travelling with uniform retardation, passes point $a$ with a speed of $30 \mathrm{~m} / \mathrm{s}$ and passes point $b$ with a speed of $20 \mathrm{~m} / \mathrm{s}$.
The distance from $a$ to $b$ is 100 m . The car comes to rest at $d$.

Find (i) the uniform retardation of the car
(ii) the time taken to travel from $a$ to $b$
(iii) the distance from $b$ to $d$
(iv) the speed of the car at $c$, where $c$ is the midpoint of $[b d]$.

## Solution

$a$ to $\left.b: \quad \begin{array}{l}u=30 \\ v=20 \\ s=100\end{array}\right\} v^{2}=u^{2}+2 a s \quad \Rightarrow \quad a=\frac{v^{2}-u^{2}}{2 s} \quad \Rightarrow \quad a=\frac{400-900}{200}$

$$
\begin{equation*}
\Rightarrow \quad a=-2.5 \mathrm{~m} / \mathrm{s}^{2} \tag{i}
\end{equation*}
$$

$\left.\begin{array}{rl}\left.a \text { to } b: \quad \begin{array}{l}u \\ v \\ v \\ a\end{array}\right) \\ =-2 \cdot 5\end{array}\right\} v=u+a t \quad \Rightarrow \quad t=\frac{v-u}{a} \quad \Rightarrow \quad t=\frac{20-30}{-2 \cdot 5}$
$a$ to $\left.d: \quad \begin{array}{l}u=30 \\ v=0 \\ a=-2 \cdot 5\end{array}\right\} v^{2}=u^{2}+2 a s \quad \Rightarrow \quad s=\frac{v^{2}-u^{2}}{2 a} \quad \Rightarrow \quad s=\frac{0-900}{-5}$
$\Rightarrow \quad s=180 \mathrm{~m} . .($ (iii)
$a$ to $\left.c: \quad \begin{array}{l}u=30 \\ a=-2 \cdot 5 \\ s=140\end{array}\right\} v^{2}=u^{2}+2 a s \quad \Rightarrow \quad v=\sqrt{u^{2}+2 a s} \Rightarrow \quad v=\sqrt{900-700}$
$\Rightarrow \quad v=\sqrt{200}=10 \sqrt{2} \mathrm{~m} / \mathrm{s}$
$\ldots(\mathrm{iv})$

## 2004 - Ordinary Level - Question 1

Three points $a, b$ and $c$, lie on a straight level road such that $|a b|=|b c|=100 \mathrm{~m}$.
A car, travelling with uniform retardation, passes point $a$ with a speed of $20 \mathrm{~m} / \mathrm{s}$ and passes point $b$ with a speed of $15 \mathrm{~m} / \mathrm{s}$.
(i) Find the uniform retardation of the car.
(ii) Find the time it takes the car to travel from $a$ to $b$, giving your answer as a fraction.
(iii) Find the speed of the car as it passes $c$, giving your answer in the form $p \sqrt{q}$, where $p, q \in \mathbb{N}$.
(iv) How much further, after passing $c$, will the car travel before coming to rest? Give your answer to the nearest metre.

## Solution

$a$ to $\left.b: \quad \begin{array}{l}u=20 \\ v=15 \\ s=100\end{array}\right\} v^{2}=u^{2}+2 a s \quad \Rightarrow \quad a=\frac{v^{2}-u^{2}}{2 s} \quad \Rightarrow \quad a=\frac{225-400}{200}$

$$
\begin{equation*}
\Rightarrow \quad a=-\frac{7}{8} \mathrm{~m} / \mathrm{s}^{2} . \tag{i}
\end{equation*}
$$

$a$ to $\left.b: \quad \begin{array}{l}u=20 \\ v=15 \\ a=-\frac{7}{8}\end{array}\right\} v=u+a t \quad \Rightarrow \quad t=\frac{v-u}{a} \quad \Rightarrow \quad t=\frac{15-20}{-\frac{7}{8}}=-5\left(-\frac{8}{7}\right)$

$$
\begin{equation*}
\Rightarrow \quad t=\frac{40}{7} \text { seconds } \tag{ii}
\end{equation*}
$$

$a$ to $\left.c: \quad \begin{array}{l}u=20 \\ a=-\frac{7}{8} \\ s=200\end{array}\right\} v^{2}=u^{2}+2 a s \Rightarrow v=\sqrt{u^{2}+2 a s} \Rightarrow \quad v=\sqrt{400+2\left(-\frac{7}{8}\right)(200)}$

$$
\begin{equation*}
\Rightarrow \quad v=\sqrt{50} \quad \Rightarrow \quad v=5 \sqrt{2} \mathrm{~m} / \mathrm{s} \tag{iii}
\end{equation*}
$$

$c$ to rest:: $\left.\quad \begin{array}{l}u=5 \sqrt{2} \\ a=-\frac{7}{8} \\ v=0\end{array}\right\} v^{2}=u^{2}+2 a s \quad \Rightarrow \quad s=\frac{v^{2}-u^{2}}{2 a} \quad \Rightarrow \quad s=\frac{0-50}{-\frac{7}{4}}$

$$
\Rightarrow \quad s \simeq 29 \mathrm{~m} . .(\mathrm{iv})
$$

## 2003 - Higher Level - Question 1(a)

The points $p, q$ and $r$ all lie in a straight line.
A train passes point $p$ with speed $u \mathrm{~m} / \mathrm{s}$. The train is travelling with uniform retardation $f \mathrm{~m} / \mathrm{s}^{2}$. The train takes 10 seconds to travel from $p$ to $q$ and 15 seconds to travel from $q$ to $r$, where $|p q|=|q r|=125$ metres.
(i) Show that $f=\frac{1}{3}$.
(ii) The train comes to rest $s$ metres after passing $r$.

Find $s$, giving your answer correct to the nearest metre.

## Solution

$\left.\begin{array}{lll}p \text { to } q: & \begin{array}{l}u=u \\ a=-f \\ t=10 \\ s=125\end{array} \\ s\end{array}\right\} s=u t+\frac{1}{2} a t^{2} \quad \Rightarrow \quad 125=10 u-50 f 1 . .$. Equation 1
Assume train continues on and comes to rest at some point $m$ :
$p$ to $\left.m: \quad \begin{array}{l}u=\frac{85}{6} \\ a=-\frac{1}{3} \\ v=0\end{array}\right\} v^{2}=u^{2}+2 a s \quad \Rightarrow \quad s=\frac{v^{2}-u^{2}}{2 a} \quad \Rightarrow \quad s=\frac{0-\frac{7225}{36}}{-\frac{2}{3}} \simeq 301$ $r$ to rest $=301-250=51$ metres

