

Applied Maths Induction Workshop 1 – Accelerated Linear Motion – Solutions

2010 – Ordinary Level – Question 1

A car travels along a straight level road.

It passes a point *P* at a speed of 12 m/s and accelerates uniformly for 6 seconds to a speed of 30 m/s.

It then travels at a constant speed of 30 m/s for 15 seconds.

Finally the car decelerates uniformly from 30 m/s to rest at a point Q.

The car travels 45 metres while decelerating.

- Find (i) the acceleration
 - (ii) the deceleration
 - (iii) |PQ|, the distance from P to Q
 - (iv) the average speed of the car as it travels from P to Q.

Solution



(iii) Distance from P to Q equals area under graph

(iv)

$$= (6 \times 12) + \left(\frac{1}{2} \times 6 \times 18\right) + (15 \times 30) + 45$$
$$= \boxed{621 \text{ metres}}$$
Average Speed
$$= \frac{\text{Total Distance}}{\text{Total Time}}$$
Firstly, must find time elapsed during deceleration phase:

$$\begin{array}{l} u = 30 \\ v = 0 \\ a = -10 \end{array} \right\} v = u + at \implies t = \frac{v - u}{a} \implies t = \frac{0 - 30}{-10} \implies t = 3 \\ \Rightarrow \quad \text{Total Time} = 6 + 15 + 3 = 24 \text{ seconds} \\ \Rightarrow \quad \text{Average Speed} = \frac{621}{24} \approx \boxed{26 \text{ seconds}} \end{aligned}$$

2007 – Ordinary Level – Question 1

A car travels from p to q along a straight level road. It starts from rest at p and accelerates uniformly for 5 seconds to a speed of 15 m/s. It then moves at a constant speed of 15 m/s for 20 seconds. Finally the car decelerates uniformly from 15 m/s to rest at q in 3 seconds.

(i) Draw a speed-time graph of the motion of the car from p to q.

- (ii) Find the uniform acceleration of the car.
- (iii) Find the uniform deceleration of the car.
- (iv) Find |pq|, the distance from p to q.
- (v) Find the speed of the car when it is 13.5 metres from p.

Solution



$$\begin{array}{c} u = 0 \\ a = 3 \\ s = 13 \cdot 5 \end{array} \right\} \quad v^2 = u^2 + 2as \quad \Rightarrow \quad v = \sqrt{u^2 + 2as} \quad \Rightarrow v = \sqrt{0 + (2)(3)(13 \cdot 5)} \Rightarrow \boxed{v = 9 \text{ m/s}}$$

A train accelerates uniformly from rest to a speed v m/s.

It then continues at this speed for a period of time and then decelerates uniformly to rest.

In travelling a total distance d metres the train accelerates through a distance pd metres and decelerates through a distance qd metres, where p < 1 and q < 1.

(i) Draw a speed-time graph for the motion of the train.

(ii) If the average speed of the train for the whole journey is $\frac{v}{p+q+b}$, find the value of b.

Solution





We know that the total distance is d. We therefore need to find the time for each section of the journey.

Acceleration

Area under graph = pd \Rightarrow $\frac{1}{2}(t_1)(v) = pd$ \Rightarrow $t_1 = \frac{2pd}{v}$ <u>Constant Speed</u> Area under graph = d - pd - qd \Rightarrow $t_2v = d - pd - qd$ \Rightarrow $t_2 = \frac{d - pd - qd}{v}$ <u>Deceleration</u> Area under graph = qd \Rightarrow $\frac{1}{2}(t_3)(v) = qd$ \Rightarrow $t_3 = \frac{2qd}{v}$ \Rightarrow Total Time = $\frac{2pd}{v} + \frac{d - pd - qd}{v} + \frac{2qd}{v} = \frac{2pd + d - pd - qd + 2qd}{v} = \frac{d + pd + qd}{v}$

$$\Rightarrow \quad \text{Average Speed} = \frac{d}{\frac{d+pd+qd}{v}} = \frac{d}{\sqrt{t}(1+p+q)} = \frac{v}{p+q+1}$$

But Average Speed =
$$\frac{v}{p+q+b} \implies b=1$$

A particle travels in a straight line with constant acceleration f for 2t seconds and covers 15 metres. The particle then travels a further 55 metres at constant speed in 5t seconds. Finally the particle is brought to rest by a constant retardation 3f.

- (i) Draw a speed-time graph for the motion of the particle.
- (ii) Find the initial velocity of the particle in terms of t.
- (iii) Find the total distance travelled in metres, correct to two decimal places.

Solution



Constant Speed

 $5tv = 55 \implies v = \frac{11}{t}$ $\underline{Acceleration}$ u = u a = f t = 2t s = 15 $v = \frac{11}{t}$ There is enough information here for two equations in u and f s = 15 $v = \frac{11}{t}$ $s = ut + \frac{1}{2}at^{2} \implies 15 = 2ut + 2ft^{2} \implies f = \frac{15 - 2ut}{2t^{2}}$ $v = u + at \implies \frac{11}{t} = u + 2ft$ $\implies 11 = ut + 2ft^{2} \implies f = \frac{11 - ut}{2t^{2}}$ $\implies 11 - ut = 15 - 2ut$ $\implies ut = 4 \implies ut = 4 \implies \dots(ii)$

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(iii) Need to find distance travelled during deceleration:

$$u = \frac{11}{t}$$

$$v = 0$$

$$a = -3f$$

$$v^{2} = u^{2} + 2as \qquad \Rightarrow \qquad s = \frac{v^{2} - u^{2}}{2a} \qquad \Rightarrow \qquad s = \frac{0 - \frac{121}{t^{2}}}{-6f}$$

$$\Rightarrow \qquad s = \frac{121}{6ft^{2}}$$
But
$$f = \frac{11 - ut}{2t^{2}} \qquad \Rightarrow \qquad f = \frac{11 - (\frac{4}{t})t}{2t^{2}} \qquad \Rightarrow \qquad f = \frac{7}{2t^{2}}$$

$$\Rightarrow \qquad s = \frac{121}{6\left(\frac{7}{2t^{2}}\right)t^{2}}$$

$$\Rightarrow \qquad s = \frac{121}{6\left(\frac{7}{2t^{2}}\right)t^{2}}$$

$$\Rightarrow \qquad s = \frac{121}{21} \approx 5.76 \text{ m}$$

$$\Rightarrow \qquad \text{Total distance travelled} \approx 15 + 55 + 5.76 = \boxed{75.76 \text{ m}}$$

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A train accelerates uniformly from rest to a speed v m/s with uniform acceleration $f m/s^2$.

It then decelerates uniformly to rest with uniform retardation $2 f \text{ m/s}^2$.

The total distance travelled is d metres.

(i) Draw a speed-time graph for the motion of the train.

(ii) If the average speed of the train for the whole journey is $\sqrt{\frac{d}{3}}$, find the value of f.

Solution



A lift starts from rest. For the first part of its descent it travels with uniform acceleration f. It then travels with uniform retardation 3f and comes to rest. The total distance travelled is d and the total time taken is t.

- (i) Draw a speed-time graph for the motion.
- (ii) Find d in terms of f and t.

Solution



Acceleration

$$\begin{array}{c} u = 0 \\ a = f \\ t = \frac{3}{4}t \end{array} \quad v = u + at \qquad \Rightarrow \qquad v = \frac{3}{4}ft \\ d = \text{Area under graph} \Rightarrow \qquad d = \frac{1}{2}t\left(\frac{3}{4}ft\right) \qquad \Rightarrow \qquad d = \frac{3ft^2}{8}$$

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A ball is thrown vertically upwards with an initial velocity of $39 \cdot 2 \text{ m/s}$.

Find (i) the time taken to reach the maximum height

(ii) the distance travelled in 5 seconds.

Solution

$$\begin{array}{c} u = 39 \cdot 2 \\ a = -9 \cdot 8 \\ v = 0 \end{array} \right\} \quad v = u + at \quad \Rightarrow \qquad t = \frac{v - u}{a} \qquad \Rightarrow \qquad t = \frac{0 - 39 \cdot 2}{-9 \cdot 8} \quad \Rightarrow \qquad \boxed{t = 4 \text{ s}} \quad \dots \text{(i)}$$

(ii) First five seconds: 4 seconds upwards, 1 second downwards.

$$\frac{\text{Upwards}}{u = 39 \cdot 2}$$

$$a = -9 \cdot 8$$

$$t = 4$$

$$s = ut + \frac{1}{2}at^{2} \implies s = (39 \cdot 2)(4) + \frac{1}{2}(-9 \cdot 8)(16) \implies s = 78 \cdot 4 \text{ m}$$

Downwards

$$\begin{array}{l} u = 0 \\ a = 9 \cdot 8 \\ t = 1 \end{array} \right\} \quad s = ut + \frac{1}{2}at^2 \qquad \Longrightarrow \qquad s = (0)(1) + \frac{1}{2}(9 \cdot 8)(1) \qquad \Longrightarrow \qquad s = 4 \cdot 9$$

 \Rightarrow Distance travelled in 5 seconds = $78 \cdot 4 + 4 \cdot 9 = 83 \cdot 3 \text{ m}$

A stone is thrown vertically upwards under gravity with a speed of u m/s from a point 30 metres above the horizontal ground. The stone hits the ground 5 seconds later.

- (i) Find the value of u.
- (ii) Find the speed with which the stone hits the ground.

Solution



 \Rightarrow 30.5 m/s in a downward direction.

Two particles P and Q, each having constant acceleration, are moving in the same direction along parallel lines. When P passes Q the speeds are 23 m/s and $5 \cdot 5 \text{ m/s}$, respectively. Two minutes later Q passes P, and Q is then moving at $65 \cdot 5 \text{ m/s}$.

Find (i) the acceleration of P and the acceleration of Q

- (ii) the speed of P when Q overtakes it
- (iii) the distance P is ahead of Q when they are moving with equal speeds.

Solution

(i) Motion of P: Motion of Q:

Overtaking occurs when $s_1 = s_2$

$$s = ut + \frac{1}{2}at^{2}$$

$$\Rightarrow s = (23)(120) + \frac{1}{2}a_{1}(120^{2})$$

$$\Rightarrow s = (5 \cdot 5)(120) + \frac{1}{2}\left(\frac{1}{2}\right)(120^{2})$$

$$\Rightarrow s = 2760 + 7200a_{1}$$

$$\Rightarrow s = 4260$$

$$\Rightarrow 2760 + 7200a_{1} = 4260$$
...when overtaking occurs
$$\Rightarrow \left[a_{1} = \frac{5}{24} \text{ m/s}^{2}\right]$$
(ii) Motion of P:

$$u = 23$$

$$a = \frac{5}{24}$$

$$v = u + at$$

$$\Rightarrow v = 23 + \frac{5}{24}(120)$$

$$\Rightarrow v = 48 \text{ m/s}$$

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(iii) Need to find the time at which $v_1 = v_2$ and then find the difference between s_1 and s_2 at this time.

Motion of
$$P$$

 $u = 23$
 $a = \frac{5}{24}$
 $t = t$
 $v = u + at$
 $t = t$
 $v = u + at$
 $t = t$
 $v = u + at$
 $v_1 = 23 + \frac{5}{24}t$
 $v_1 = v_2$
 $\Rightarrow 23 + \frac{5}{24}t = 5 \cdot 5 + \frac{1}{2}t$
 $\Rightarrow 23 + \frac{5}{24}t = 5 \cdot 5 + \frac{1}{2}t$
 $\Rightarrow 552 + 5t = 132 + 12t$
 $\Rightarrow 7t = 420$
 $\Rightarrow t = 60$
 $s_1 = (23)(60) + \frac{1}{2}(\frac{5}{24})(60^2)$
 $s_2 = (5 \cdot 5)(60) + \frac{1}{2}(\frac{1}{2})(60^2)$
 $s_2 = 1230$

 \Rightarrow *P* is 525 metres ahead of *Q* when they are travelling with equal speeds.

Car A and car B travel in the same direction along a horizontal straight road. Each car is travelling at a uniform speed of 20 m/s.

Car A is at a distance of d metres in front of car B.

At a certain instant car A starts to brake with a constant retardation of 6 m/s^2 .

0.5 s later car *B* starts to brake with a constant retardation of 3 m/s^2 .

Find (i) the distance travelled by car A before it comes to rest.

(ii) the minimum value of d for car B not to collide with car A.

Solution

(i) Car A

$$u = 20$$

$$a = -6$$

$$v = 0$$

$$v^{2} = u^{2} + 2as \implies s = \frac{v^{2} - u^{2}}{2a} \implies s = \frac{0 - 400}{-12} \implies s = \frac{100}{3} \text{ m}$$
(i) Car B
First 0.5 seconds Afterwards

$$u = 20$$

$$t = 0.5$$

$$a = 0$$

$$s = ut + \frac{1}{2}at^{2}$$

$$u = 20$$

$$a = -3$$

$$v = 0$$

$$s = \frac{v^{2} - u^{2}}{2a}$$

$$\Rightarrow s = (20)(0.5) \implies s = \frac{0 - 400}{-6}$$

$$\Rightarrow s = 10 \implies s = \frac{200}{3}$$

$$\Rightarrow \text{ Overall, it takes car } B\left(10 + \frac{200}{3}\right) = \frac{230}{3} \text{ metres to stop.}$$

$$\Rightarrow d \text{ must be at least } \frac{230}{3} - \frac{100}{2} = \boxed{\frac{130}{3} \text{ metres}}$$

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2008 – Ordinary Level – Question 1

Four points *a*, *b*, *c* and *d* lie on a straight level road.

A car, travelling with uniform retardation, passes point a with a speed of 30 m/s and passes point b with a speed of 20 m/s.

The distance from a to b is 100 m. The car comes to rest at d.

- Find (i) the uniform retardation of the car
 - (ii) the time taken to travel from a to b
 - (iii) the distance from b to d
 - (iv) the speed of the car at c, where c is the midpoint of [bd].

Solution

| a to b : | $ \begin{array}{l} u = 30 \\ v = 20 \\ s = 100 \end{array} \right\} v^2 = u^2 + 2as $ | ⇒ | $a = \frac{v^2 - u^2}{2s}$ | $\Rightarrow a = b$ | $\frac{400-900}{200}$ |
|------------|--|---------------|----------------------------|-------------------------------|-------------------------------|
| | | | \Rightarrow | $a = -2 \cdot 5 \mathrm{m}$ | /s ² (i) |
| a to b : | $ \begin{array}{l} u = 30 \\ v = 20 \\ a = -2 \cdot 5 \end{array} \right\} v = u + at $ | \Rightarrow | $t = \frac{v - u}{a}$ | $\Rightarrow t = \frac{2}{2}$ | $\frac{20-30}{-2\cdot 5}$ |
| | | | \Rightarrow | t = 4 secon | ds(ii) |
| a to d : | $ \begin{array}{l} u = 30 \\ v = 0 \\ a = -2 \cdot 5 \end{array} \right\} v^2 = u^2 + 2as $ | ⇒ | $s = \frac{v^2 - u^2}{2a}$ | \Rightarrow $s = -$ | $\frac{0-900}{-5}$ |
| | | | \Rightarrow | <i>s</i> = 180 m | (iii) |
| a to c : | $ \begin{array}{c} u = 50\\ a = -2 \cdot 5\\ s = 140 \end{array} \right\} v^2 = u^2 + 2as $ | ⇒ | $v = \sqrt{u^2 + 2as}$ | $\Rightarrow v = \cdot$ | √900 <i>−</i> 700 |
| | | | \Rightarrow | $v = \sqrt{200} =$ | $10\sqrt{2} \text{ m/s}$ (iv) |

2004 – Ordinary Level – Question 1

Three points a, b and c, lie on a straight level road such that |ab| = |bc| = 100 m.

A car, travelling with uniform retardation, passes point a with a speed of 20 m/s and passes point b with a speed of 15 m/s.

- (i) Find the uniform retardation of the car.
- (ii) Find the time it takes the car to travel from a to b, giving your answer as a fraction.
- (iii) Find the speed of the car as it passes c, giving your answer in the form $p\sqrt{q}$, where $p, q \in \mathbb{N}$.
- (iv) How much further, after passing c, will the car travel before coming to rest? Give your answer to the nearest metre.

Solution

| a to b : | $ \begin{array}{c} u = 20 \\ v = 15 \\ s = 100 \end{array} \right\} v^2 = u^2 + 2as \qquad \Longrightarrow \qquad a = \frac{v^2 - u^2}{2s} $ | $\Rightarrow \qquad a = \frac{225 - 400}{200}$ |
|------------------------|---|--|
| | \Rightarrow | $a = -\frac{7}{8}$ m/s ² (i) |
| a to b : | $ \begin{array}{l} u = 20 \\ v = 15 \\ a = -\frac{7}{8} \end{array} v = u + at \Longrightarrow t = \frac{v - u}{a} \qquad \Longrightarrow \qquad$ | $t = \frac{15 - 20}{-\frac{7}{8}} = -5\left(-\frac{8}{7}\right)$ |
| | \Rightarrow | $t = \frac{40}{7}$ seconds(ii) |
| <i>a</i> to <i>c</i> : | $ \begin{array}{l} u = 20 \\ a = -\frac{7}{8} \\ s = 200 \end{array} \right\} v^{2} = u^{2} + 2as \implies v = \sqrt{u^{2} + 2as} \implies v = \sqrt$ | $v = \sqrt{400 + 2\left(-\frac{7}{8}\right)(200)}$ |
| | $\implies \qquad v = \sqrt{50} \qquad \implies \qquad$ | $v = 5\sqrt{2} \text{ m/s} \dots (\text{iii})$ |
| c to rest:: | $ \begin{array}{c} u = 5\sqrt{2} \\ a = -\frac{7}{8} \\ v = 0 \end{array} \right\} v^2 = u^2 + 2as \Longrightarrow \qquad s = \frac{v^2 - u^2}{2a} $ | $\implies \qquad s = \frac{0-50}{-\frac{7}{4}}$ |
| | \Rightarrow | $s \simeq 29 \text{ m} \dots (\text{iv})$ |

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The points p, q and r all lie in a straight line.

A train passes point p with speed u m/s. The train is travelling with uniform retardation f m/s². The train takes 10 seconds to travel from p to q and 15 seconds to travel from q to r, where |pq| = |qr| = 125 metres.

(i) Show that $f = \frac{1}{3}$.

(ii) The train comes to rest s metres after passing r. Find s, giving your answer correct to the nearest metre.

Solution



Assume train continues on and comes to rest at some point m:

$$\begin{array}{c} u = \frac{85}{6} \\ p \text{ to } m: & a = -\frac{1}{3} \\ v = 0 \end{array} \right\} \quad v^2 = u^2 + 2as \qquad \implies \qquad s = \frac{v^2 - u^2}{2a} \qquad \implies \qquad s = \frac{0 - \frac{7225}{36}}{-\frac{2}{3}} \approx 301 \\ r \text{ to rest} = 301 - 250 = \boxed{51 \text{ metres}} \end{array}$$

 $P_{age}17$