

Question 3

(25 marks)

- (a) The mean lifetime of light bulbs produced by a company has, in the past, been 1500 hours. A sample of 100 bulbs, recently produced by the company, had a mean lifetime of 1475 hours with a standard deviation of 110 hours. Test the hypothesis that the mean lifetime of the bulbs has not changed, using a 0.05 level of significance.

Solution:

$$H_0: \mu = 1500$$

$$H_1: \mu \neq 1500$$

$$Z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{1475 - 1500}{\frac{110}{\sqrt{100}}} = -2.28$$

As $-2.28 < -1.96$, we reject H_0 at the 5% level of significance.

(could also use a confidence interval as in Q 2(a))

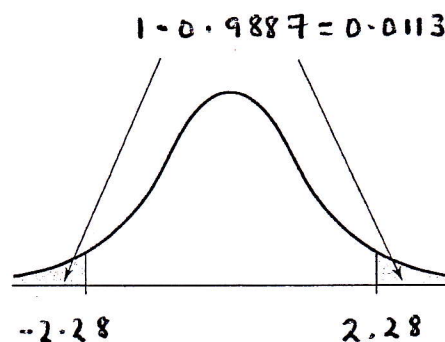
- (b) Find the p-value of the test you performed in part (a) above and explain what this value represents in the context of the question.

Solution:

If $p \leq 0.05$ reject H_0 .

If $p \geq 0.05$ fail to reject H_0 .

$$p\text{-value} = 2(0.0113) \\ = 0.0226$$



$$0.0226 < 0.05$$

Thus, there is strong evidence to reject H_0 .

Therefore, we conclude there is strong evidence to reject the claim that the mean lifetime of the bulbs has not changed.

Note: The meaning is that if the null hypothesis is true, we would expect to see data as or more extreme than we got 2.26% of the time (more extreme here means a more extreme sample mean, i.e. a sample mean as far or further from 1500 than 1475, so this means a sample mean ≤ 1475 or ≥ 1525).

"when p is low H_0 must go"